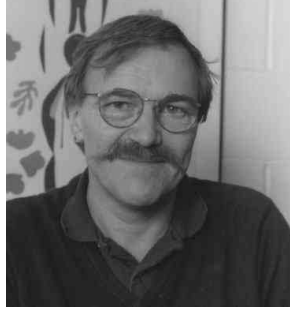# CLASSICAL CEPHEIDS – A REVIEW

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Abstract

We briefly review the evolutionary status of the classical Cepheid variables, their structure, and the properties of the linear modes. Then we discuss the current status of the nonlinear hydrodynamical modelling, including modern adaptive mesh techniques, and the sensitivity to opacity. We stress the importance of resonances among the normal modes of pulsation because they can be used to impose severe constraints on the models and on the input physics.

We show that recent hydrodynamics computations provide an explanation for the hitherto unexplained dip in the histogram of Galactic Cepheids around 8–10^d.

The EROS and MACHO observations of classical Cepheids in the Large and Small Magellanic Clouds have opened up a new exciting perspective on these variable stars. This new observational basis is very important, not only because of the sheer number of new Cepheids, but especially because the composition of the MCs is known to differ from our Galaxy's, thus extending the range of astrophysical parameters. These new data have been found not to be fully compatible with our current Cepheid modelling and are forcing a review of our assumptions and input physics.

1 INTRODUCTION

This Colloquium to within a month marks the fourth centennial of variable star Astronomy. Indeed, in 1596, at the time of Tycho Brahe, and a dozen years before the development of the telescope, a priest in Friesland, David Fabricius, recorded that a star in the constellation of the Whale that was quite bright in August had disappeared from sight by the end of the year, only to reappear with full brightness the following year. Because of its unusual nature this star was called Stella Mirabilis (the 'remarkable star') or Mira for short, and it is now a prototype of a whole class of variable stars. Almost another 200 years elapsed before a young Englishman, John Goodricke, discoved the variability of δ Cephei, which came to lend its name to the class of variable stars which interest us here. Cepheids are perhaps the best known variable stars because of the relatively tight relation between period and luminosity that Leavitt discovered in 1912, and which catapulted the Cepheids into the prominent cosmological role of distance indicators.

2   STRUCTURE AND LINEAR PROPERTIES

Cepheids are stars that have evolved away from the Main Sequence. The evolutionary tracks for stars in the Cepheid mass range typically cross the *instability strip* three times (*e.g.* [1], [2]). In this instability strip the stars are vibrationally unstable and exhibit pulsations. Generally the first crossing is ignored as far as Cepheid models are concerned. The rationale is that, since the evolution is much more rapid there than on the subsequent horizontal loops, there is a very low probability of observing Cepheids on the first crossing. However, the large number of Cepheids that are being observed in the Magellanic Clouds make the possibility of detection increasingly likely. A search for such first crossers could yield interesting new information about the structure of the Cepheid variables.

Evolutionary calculations completely ignore pulsation because the pulsation is essentially a surface phenomenon - see below - that cannot have a large effect on the evolution. On the other hand, because the (nuclear) evolution time scale is very much longer than the dynamical or thermal time scales the pulsation studies can in turn ignore evolution. As far as pulsation studies are concerned, evolution calculations merely provide information about the location of the loops and thus about the luminosities of the stars as a function of mass, the so-called Mass–Luminosity (M–L) relation.

That the Cepheid pulsation is a surface phenomenon is illustrated on the left of Fig. 1. The top shows the pressure and temperature runs of a Cepheid model (M=$6M_\odot$, L=$4000L_\odot$, Z=0.01, X=0.70) in terms of the normalized radius. The pins point to the ionization fronts, *i.e.* the points where the relative mass fraction of the indicated ion is 50%. The very sharp drop in temperature is caused by the hydrogen ionization front and is notorious for the numerical headaches it has caused in the hydrodynamics.

The bottom graph displays the modulus of the absolute radial displacement eigenvector (scaled by the stellar radius) for the fundamental mode and the first two overtones, having 0, 1 and 2 minima, respectively. The bars on top indicate the exterior mass fraction. While the outer 60% of the radius partakes in the pulsation, it is less than 1% of the mass that is affected.

The right side of Fig. 1 shows the work integrands for the fundamental mode and first overtone. The driving which occurs in the partial ionization regions ([3]) is seen to be localized to the outer $\sim$0.01% of the mass, or 10% of the radius.
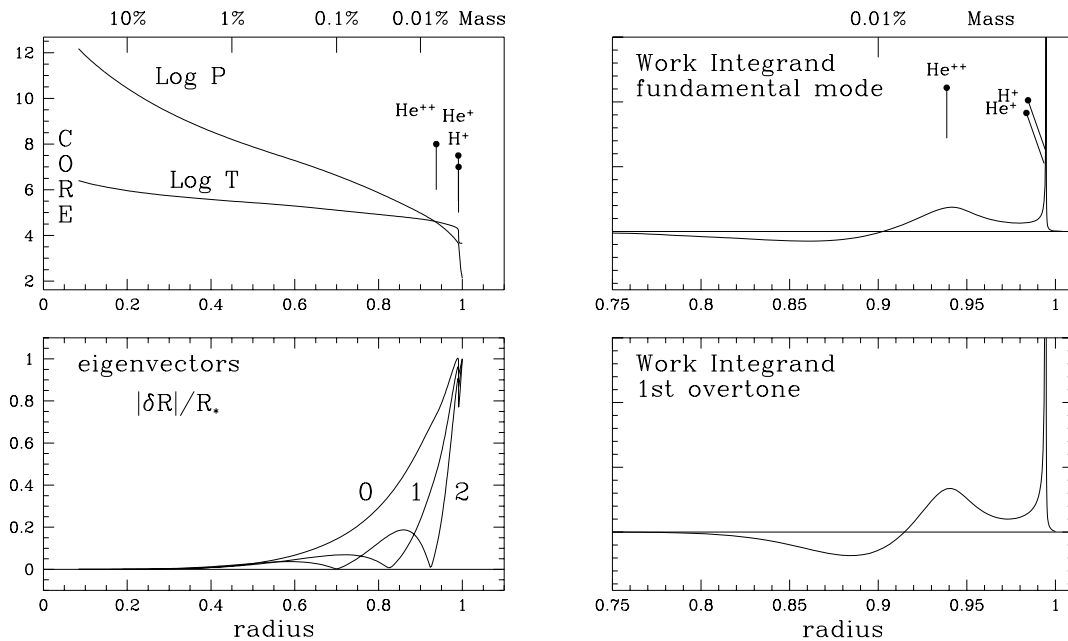

Fig. 1 *left*: Pressure and temperature profile, eigenvectors *vs* normalized radius, and *vs* exterior mass (top scale); *right*: work integrands (cycle averaged $< pdV >$).

3 NONLINEAR PULSATIONS

3.1 Bump Progression and Fourier decomposition

Resonances are known to play an important role in shaping the morphology of the light- and radial velocity curves. In 1926 Hertzsprung noted that in the vicinity of a period of 10^d the fundamental mode Cepheid light curves exhibit a secondary maximum or shoulder that moves from descending to ascending branch as the period increases. This bump progression is even more striking [4] in the phases $\phi_{k1} = \phi_k - k\phi_1$ and amplitude ratios $R_{k1} = A_k/A_1$ of the Fourier decomposition

$$f(t) = A_0 + \sum_{k=1,\ldots} A_k \cos(k\omega t + \phi_k) = A_0 + A_1 \left(\sum_{k=1,\ldots} R_{k1} \cos(k\omega t' + \phi_{k1}) \right) \tag{1}$$

($t' = t - t_o$, where the epoch t_o is such that $\phi_1 = 0$). The advantage of these *relative* quantities ϕ_{k1} and R_{k1} is that the decomposition becomes independent of the origin of time and of the overall pulsation amplitude. On the basis of a comparison of linear models and hydrodynamics Simon & Schmidt [5] conjectured that the bump progression is the result of a 2:1 resonance ($2\omega_0 \approx \omega_2$) between the fundamental mode of pulsation and the second overtone.

The presence of a resonance such as this puts a severe constraint on the modelling, a crude sort of asteroseismology. In fact, for a number of years Cepheid modelling was plagued by a so-called 'mass discrepancy', which was the inability of evolutionary M–L relations to put the 2:1 resonance at the observed period of 10^d. Simon's [6] bold suggestion that the opacities had to be wrong initiated a large revision of the opacity tables [7] [8]. Calculations with these new opacities, supplemented with the low temperature ones of [9], showed essential agreement between evolution, linear pulsation theory and observations [10][11] for the Galactic Cepheids. This is one of those remarkable occasions of useful positive feedback between astrophysics and physics.

3.2 Numerical Hydrodynamics

Numerical hydrodynamical modelling goes back to Christy in the 1960s. These early calculations reproduced the bulk properties of the Galactic Cepheids reasonably well, except for the mass discrepancy. With the new opacities, the complete agreement between the observations and the hydrodynamical computations is good, both for the light- and for the radial velocity curves [10].

While numerical hydrodynamics reproduce the bump and the Fourier coefficient progression quite well, they provide no real explanation for the underlying mechanism. For this purpose the general amplitude equation formalism was developed (for a review and references *cf.* [12] [13]). These amplitude equations describe the nonlinear interaction of the excited modes and give the behavior of the Fourier coefficients as a function of period.

3.3 Amplitude Equations

This semi-analytical formalism takes advantage of the two small parameters with which the Cepheids are endowed, namely (1) the ratio η of growth rate to frequency for the excited mode(s) and (2) the weak anharmonicity of the pulsations (which allows an expansion in the pulsation amplitude). Together they allow one to find a beautifully simple description of the pulsation.

The formalism has been described in detail elsewhere [13]. Here, as an illustration, we would like to just mention the simplest possible situation, namely that of a single excited mode. Let its linear frequency and growth rate be ω and κ. Furthermore, let A denote the pulsation amplitude. The amplitude equation which governs the behavior of the pulsation is

$$\frac{dA}{dt} = \kappa A - QA^3 + O(A^5), \tag{2}$$

where Q is a nonlinear quantity that depends on the structure of the star.

If the star is linearly stable, $\kappa < 0$ and the long term solution of the equation is $A(t) \to 0$. In the case of linear instability, on the other hand, the star approaches a constant pulsation amplitude given by $A_{lc} = \sqrt{\kappa/Q}$, *i.e* a limit cycle pulsation. (For simplicity we have assumed that all the other modes are linearly stable.) Finally, all stellar variables, such as radius, velocity, temperature, luminosity, *etc.*, are expressible in terms of A_{lc}, *e.g.*

$$m_*(t) - <m> = A\cos\bar{\omega}t + \left((*)A^2 + (*)A^2\cos2\bar{\omega}t \right) + O(A^3). \tag{3}$$

Here $\bar{\omega} = \omega + $ a nonlinear correction. The formalism generates the successive terms of a Fourier expansion. This is obviously a useful property since such an expansion is the natural way to quantify the light curve as we have seen in Eq. 1.

Of course, the formalism is more interesting and useful when several modes are coupled, and in particular when they are coupled through a resonance [12] [13]. Thus, for example, amplitude equations appropriate for the resonant coupling $(2\omega_0 \approx \omega_2)$ of the fundamental mode with the second overtone provide an excellent description and explanation of the light- and radial velocity curves of the bump Cepheids throughout the whole resonance region.

3.4   Effect of the Metallicity

Because of the difference in metallicity between the Galaxy and the Magellanic Clouds the behavior of the light curves as a function of Z is of particular interest. In Fig. 2. we show some preliminary results of a survey of fundamental Cepheid pulsations (Piciullo, Buchler, Kolláth & Goupil, in progress). The top left figure displays the behavior of the magnitude Fourier phases as a function of nonlinear period for three values of metallicity Z for fixed X=0.7 (The lines are merely there to visually connect the computed points). The M–L relations (with slope 1/3.56) for all three sequences have been chosen to have their 2:1 resonance at 10^d, and the sequences run parallel to the blue edge, offset by 100 K. It is noteworthy that in the resonance region the metallicity has a strong effect on the phases and on the amplitude ratios. This sensitivity shows that care must be exercised when the observational Fourier decomposition coefficients are used to locate the resonance.
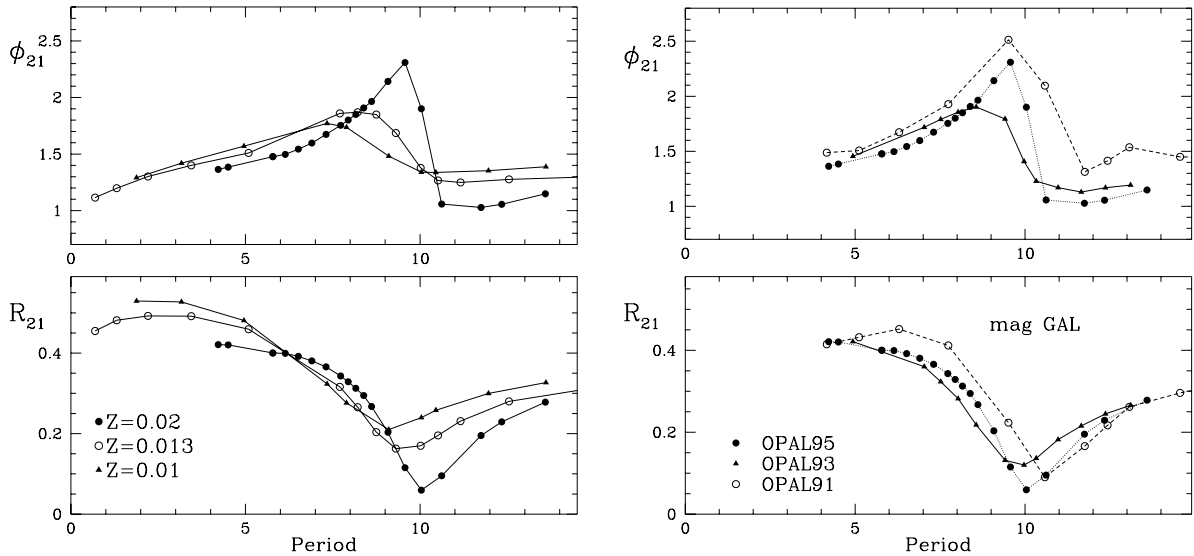

Fig. 2 Fourier decomposition coefficients, *left*: Effect of metallicity; *right*: Effect of opacity.

The observational data for the Galaxy [4] show a considerably larger scatter above $\approx 8^d$. Some of this spread is due to the finite width of the instability strip, and to a spread in metallicity, with the latter clearly dominating. Taken in connection with the results of Fig. 2 *left*, this could be interpreted as indicating that the Galaxy has a large dispersion in metallicity.

3.5   Missing Fundamental Cepheids?

Fig. 3 addresses the stability of the fundamental *limit cycles*. Shown here is the Floquet stability coefficient corresponding to a perturbation with the first overtone for a sequence that runs 100K to the left of the instability strip. The figure clearly indicates that *the fundamental limit cycle is unstable* in the range from ≈ 8–10^d for all these sequences with $Z = 0.02$. This result is largely independent of the opacity, in fact the models of [10] (OPAL91) were also unstable. Preliminary results indicate that for sequences located further away from the blue edge the instability of the limit cycle decreases, but only slightly. These results suggest that there should be no or few fundamental pulsators with $Z = 0.02$ in the period range ≈ 8–10^d.

What does this then mean? It turns out that the first overtone limit cycle is stable in this regime, so that these stars must instead be first overtone pulsators with periods ≈ 5.6–7^d. Histograms of *all* observed Galactic Cepheids and of Andromeda [2] indeed show a deficiency of Cepheids short of the 10^d period, and a corresponding increase short of 7^d (This deficiency was left as an unexplained puzzle in ref. [2]). However, some fundamental Cepheids are observed in the 8–10^d range. These stars should therefore have lower Z values such as to make the fundamental limit cycles stable. (The corresponding histograms for the MCs do not display these features, in agreement with their lower overall metallicity). These results corroborate our suggestion above that the Galactic Cepheids have a sizeable metallicity dispersion. An observational determination of the metallicity as a function of period of both the fundamental and the first overtone Galactic Cepheids would be an interesting test of these theoretical predictions and a further check for the accuracy of the opacities.
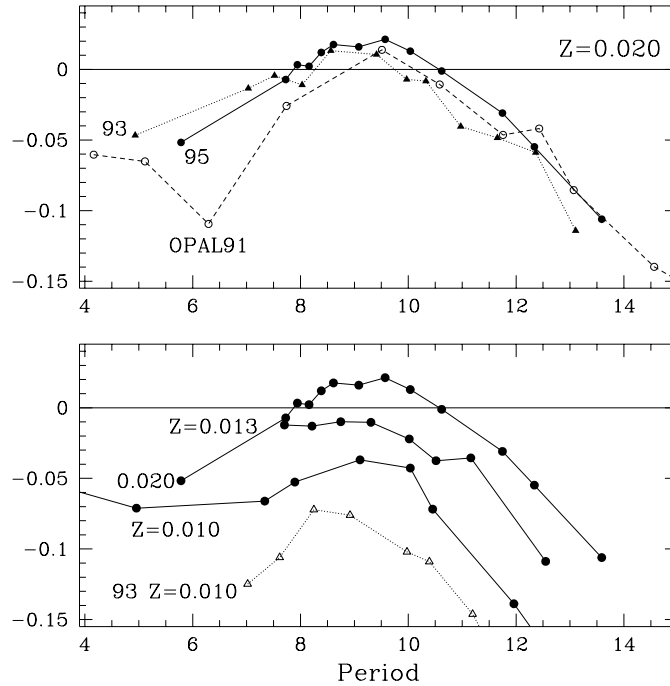

Fig. 3 Linear stability (Floquet coefficient) of fundamental limit cycle; *top:* effect of opacity,   *bottom:* effect of Z.

3.6   Uncertainties in the Opacities

How sensitive are the nonlinear pulsations to the remaining uncertainties in the opacities? The right side of Fig. 2 gives a partial answer to this question. We show here the lowest Fourier parameters for three sequences of models. The two sequences, marked with triangles (OPAL93) and dots (OPAL95) were both computed for a sequence whose M–L relation was adjusted so that the 2:1 resonance occurs at 10^d (with a slope of 1/3.56). The open circle sequence that has its resonance at $\approx 10^d.5$ represents sequence A of ref. [10] which was computed with the first (1991) OPAL revision of the opacities.

Our computations show that overall the effect of the change from OPAL93 to OPAL95 is roughly equivalent to an increase of Z from 0.02 to 0.03 (The reason is that the sensitivity is mostly due to the iron peak). This trend is also consistent with the results shown in Fig. 2.

3.7 Overtone Cepheids

The modelling situation is less comfortable for the first overtone (s Cepheid) light curves. The observational Fourier decomposition coefficients display a very large excursion (Z shape) in ϕ_{21} near 3^d. This unambiguously indicates that a resonance is active in this region. Is was conjectured by Antonello & Poretti [14] that the structure is a result of a 2:1 resonance of the excited first overtone with the fourth overtone. However numerical hydrodynamical calculations [15] [16] have failed to reproduce the observed features. One of the problems seems to be the fact that the fourth overtone is very strongly damped, and thus hard to excite through nonlinear resonant coupling. It would certainly help theorists decipher the problem if first overtone radial velocity data could be obtained and be Fourier decomposed.

3.8 Beat Cepheids

The so-called beat Cepheids are observed to pulsate in two modes simultaneously, most of them in the fundamental and the first overtone (a few also in the first and second overtones), with constant amplitudes and phases. They are observed both in the Galaxy and in the MCs. Hydrodynamics codes have failed so far to produce beat behavior in the observed period range. For this behavior to occur both the fundamental and the first overtone limit cycles must exist, but must be unstable (*e.g.* [13]). Numerical simulations with radiative codes show that this destabilization does not occur unless some resonance destabilizes the cycles. The cause for beat behavior remains a puzzle at the present time.

4 RESONANCES

We have already discussed the dynamical effects of the bump resonance near 10^d. Are there other resonances that play a dynamical role? In order to examine the presence of resonances we have computed a set of radiative Cepheid models with masses ranging from 2 to $8M_\odot$, each with 9 M–L relations and effective temperatures varying from 7400–4600K. In Fig. 4 we display the linear period ratios of the middle M–L relation ($\mathrm{Lg}L = 3.56\mathrm{Lg}M + 0.775$) with $Z=0.02$. Only those models are displayed in the left (right) graph for which the fundamental (first overtone) is linearly unstable. The reason for only showing unstable models is that this is a necessary (but not sufficient) condition for the existence of a fundamental (first overtone) limit cycle. Furthermore, for the top curves (P_{10} (P_{21}), because of their usefulness for beat Cepheids we have only included those models which, in addition, are unstable in the first (second) overtone.

The 'bump' resonance ($P_{20}=1/2$) for the fundamental Cepheids clearly stands out in the left side figure at 10^d, as does the resonance ($P_{40}=1/3$) near 7^d which also plays a dynamical role, albeit much less important [13]. Finally we note that there are further resonances, 4:1 and 5:1 at lower period, but with much higher overtones. These modes could play a dynamical role because the linear stability is not a monotonically increasing function of mode number [20].

At the right edge of the figure is the half-integer resonance $P_{10}=2/3$ which gives rise to pulsations that are periodic, but repeat only every other cycle (period-two behavior) [22]. A recent analysis of Antonello suggests that the star CC Lyr might well exhibit this behavior. The data are very limited and additional observations of this star would be extremely useful, especially if they could confirm the theoretical prediction [17] that some Cepheids in the 20–25^d period range might undergo regularly alternating cycles.

The Galactic beat Cepheids have a narrow range of period ratios, decreasing from $P_{10}=0.7097$ for TU Cas ($P_0=2^d.1393$) to 0.6967 for V367 Sct ($P_0=6.2929$). Both the range and the trend are consistent

with the results of Fig. 4.

In the right hand graph we show the period ratios relevant for first overtone pulsators. Again one sees immediately that the observed period ratio $P_{21}=0.8008$ for the star CO Aur at $P_1=1.7830^d$ is in good agreement with the linear period ratios. Of particular interest is the resonance $P_{41}=1/2$ near 3^d that was already mentioned in connection with the s Cepheids [14]. As for the fundamentals there are also further, higher order resonances at lower period. Again these resonances can solely play a dynamical role if the corresponding overtone is only slightly damped, which seems to occur around the 9th overtone[20].

For lack of space we have only shown the period ratios for one M–L relation. It is obvious that these ratios display some sensitivity to M–L. Roughly speaking, a higher M–L relation makes the period ratios drop more rapidly with P_0.
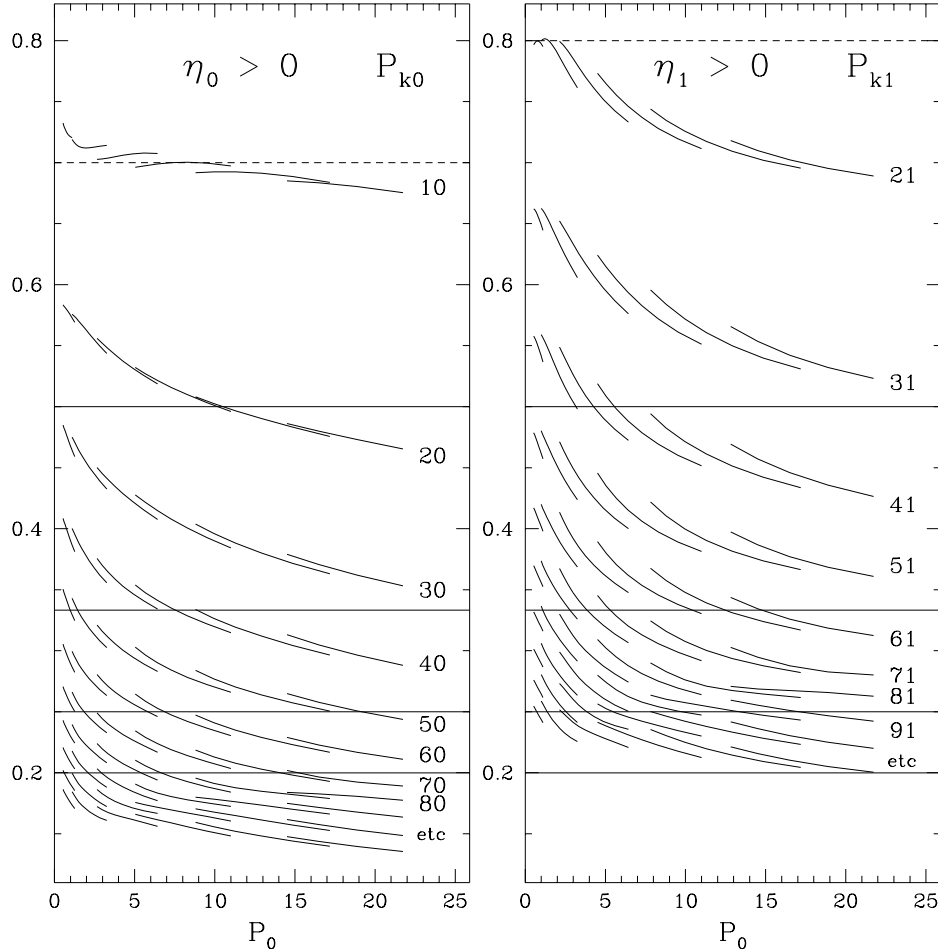

Fig. 4 Linear period ratios, top to bottom P_{10} to $P_{11\,0}$ and P_{21} to $P_{11\,1}$ for $X=0.7$, $Z=0.02$.

We have only considered two mode resonances here. It does not appear at present that multi mode resonances play a role in Cepheids (see however [23] for RR Lyrae).

4.1   Adaptive Codes

Lagrangean codes have always been hampered by the fact that the hydrogen ionization front is extremely narrow and that it undergoes substantial motion through the star during the pulsation. The resultant poor spatial resolution has given rise to inaccuracies, as well as to bad jitter, especially in the light curves. In the last few years a number of groups have made use of modern numerical adaptive mesh techniques [24] [25] [26] to provide a good spatial resolution throughout the pulsation cycle. In

addition these codes incorporate an option to solve the radiation hydrodynamics equations instead of the usual radiation equilibrium diffusion equation.

The new codes provide much smoother and more accurate light curves. However, for the Cepheids and RR Lyrae, at least, the global properties, such as the Fourier decomposition coefficients fortunately are not all that different. The same cannot be said for the Pop. II Cepheids where these numerical improvements are essential.

4.2 Convection

Christy early on estimated that convection should be inefficient and the convective flux should be small in the broad vicinity of the blue edge. In contrast, near the read edge convection must be essential and in fact it determines the location of the red edge. Convection has generally been ignored in Cepheid models. Recently Stellingwerf and Bono have advocated and used a turbulence/convection recipe that seems to give quite reasonable results (*cf.* Bono in this Volume).

4.3 Strange Modes

A very careful look at Fig. 4 shows some irregularities in the behavior of the period ratios for the higher overtones. A thorough look has indicated that this is not a numerical problem, but that it has a physical origin. In Fig. 5 we show a blow-up for a typical example of a $M{=}5M_\odot$, $L{=}4348L_\odot$, $Z{=}0.02$ sequence (with OPAL95). Interestingly and somewhat surprisingly, the figure displays a level crossing that is quite familiar from 'strange' (radial) modes [18] in luminous models, and from nonradial modes when g and p modes overlap [19]. Here, it is due to the appearance of a strange mode. One can show that *this mode has no direct counterpart in the nonadiabatic vibrational spectrum.* Interestingly this strange mode is generally either linearly unstable or almost unstable. Such a behavior was uncovered some time ago [20], but the strange nature of the mode was not realized at the time. A detailed study of these strange modes in Cepheid models will be made in [21].
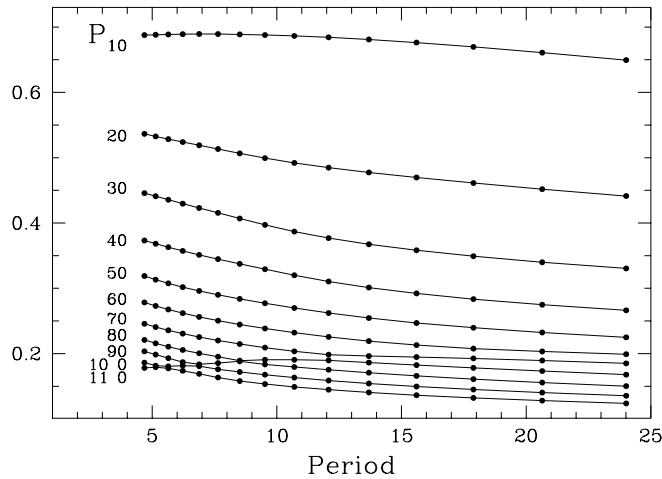

Fig. 5 Period ratios $P_{k0} = P_k/P_0$ *vs* P_0 for a $M{=}6M_\odot$, $L{=}4348L_\odot$ model, exhibiting the presence of a strange mode.

Does the presence of this strange mode have any dynamical and observable consequences? Figs. 5 and 4 show that the strange mode can be in a 5:1 resonance with the fundamental mode, or in a 4:1 resonance with the first overtone, perhaps simultaneously. Because of its marginal stability and excitability it could therefore play a role in destabilizing the limit cycle. How resonances can destabilize the limit cycle is nicely illustrated in Fig. 1 of [22]. This is clearly of interest in connection with the so far unexplained beat behavior in Cepheids. We have undertaken a systematical numerical hydro survey to explore this conjecture.

5   MAGELLANIC CLOUD CEPHEIDS

We have pointed out above that the new opacities have seemingly reconciled stellar evolution and pulsation with the observations [10] [11] for the *Galactic* Cepheids. However the new MC observations have recreated havoc.

The EROS and MACHO projects have provided a plethora of Cepheid data in the Magellanic Clouds (see this Volume). The fact that the MC are metal deficient by factors of 2 to 4 compared to the Galaxy makes these results particularly interesting for understanding Cepheids.

In order to show the global effects of a reduced metallicity we have also computed the linear properties of the same model sequences as in Fig. 4 but with $Z=0.01$, more appropriate for the average metallicity of the LMC. The decrease in Z causes an appreciable upward shift of all the P_{k0}, but a much smaller shift in the P_{k1}. The reason for this difference in shift is that the metal opacities affect mostly the structure at high temperatures ($>100\,000$K) where only the fundamental mode has a sufficiently large amplitude. The general trends of the shifts are in agreement with observations, for example, the beat Cepheids have higher period ratios in the MCs (*cf.* Welch in this Volume; see also [27])). Unfortunately, however, the beat Cepheid data do not put any serious constraints on the M–L relations and on the models [28].

A systematic and specific comparison for the bump Cepheids brings out a severe problem [28]. Most damning is the fact that the bump resonance appears in approximately the same period range 9–11^d and at approximately the same luminosity in the LMC and the SMC. This implies very low masses, especially in the metal deficient SMC. It also requires M–L relations that do not seem reconcilable with stellar evolution calculations (and the current OPAL opacities).

6   CONCLUSION

The dark matter searches have provided us with wonderful new data on thousands of variable stars in galaxies which are endowed by different metallicities. The good news is that these data provide us with an enormously broadened data base, but the bad news is that theorists have been temporarily sent back to the drawing board.

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